

Kerr-Schild Riemannian acoustic black holes in dynamo plasma laboratory

by

L.C. Garcia de Andrade

Departamento de Física Teórica – IF – Universidade do Estado do Rio de Janeiro-UERJ
Rua São Francisco Xavier, 524
Cep 20550-003, Maracanã, Rio de Janeiro, RJ, Brasil
Electronic mail address: garcia@dft.if.uerj.br

Abstract

Since Alfven, dynamo and sound waves and the existence of general relativistic black holes are well established in plasma physics, this provides enough motivation to investigate the presence of acoustic black-hole effective metric of analogue Einstein's gravity in dynamo flows. From nonlinear dynamo equations, one obtains a non-homogeneous wave equation where it is shown that the non-homogeneous factor is proportional to time evolution of the compressibility factor. In the Navier-Stokes case for a finite Reynolds number the acoustic black-holes also exists on the stretching plasma flows. In the magnetostatic case the dynamo is marginal. A coupled nonlinear plasma flow solution is found for the dynamo equation where the effective black hole solution of the scalar effective equation yields an imaginary part of the growth of magnetic field. Therefore though the real part of the growth rate of the magnetic field is negative or null, since there is a temporal oscillation in magnetic field, the solution represents a slow dynamo. Thus acoustic black holes are shown to definitely contribute to dynamo action of the effective plasma spacetime. It is suggested that a fast dynamo effective spacetime may also contain an acoustic black hole. In the case of planar waves the effective metric can be cast in Kerr-Schild spacetime form. The Killing symmetries are explicitly given in this metric and the growth of dynamo waves. A further example of effective spacetime is given in collision plasmas. **PACS numbers:** 02.40.Hw:differential geometries. 91.25.Cw:dynamo theories.

I Introduction

Several attempts have very recently been made in building an artificial black hole in laboratory. In particular optical black holes [1] have been obtained in optical fibers. Acoustic black holes are in general obtained as effective spacetime metric [2] on a fluid media or flow. As in optical black holes a physical medium able to support some kind of waves are used as motivations to derive the acoustic or optical effective black holes. Since plasma medium is able to support this kind of waves along with Alfvén waves and dynamo waves, it seems worth enough to investigate in this paper the effective metric of acoustic black holes in plasmas. Stretching of the magnetic field is in generally considered a fundamental ingredient for the dynamo [3] existence. In this report however one shows that acoustic effective black holes analogue can be obtained on a non-dynamo dissipative plasma flow rectangular slab. These so-acoustic black holes are effective analogue pseudo-Riemannian metrics given by the homogeneous wave equation from linearised Euler flows. Recently non-Riemannian vortex acoustic metrics [4] in Navier-Stokes [5] flows have been investigated by the author. In this paper one also show that the non-Riemannian geometry is not actually needed if one considers the pseudo-Riemannian acoustic spacetime non-homogeneous wave. Is exactly this non-homogeneous or non-Riemannian factor which yields the non-compressibility of the flow. Since is exactly this compressibility which is actually connected with stretching one may say that acoustic black holes would be obtained in stretching dissipative non-Riemannian flows while non-dissipative, either incompressible or compressible acoustic black holes could be naturally obtained. Magnetostatic fields are obtained in plasma dissipative flows. The stretching of the plasma flow is computes as well. Therefore Riemannian non-Riemannian manifolds can be used in this way to investigated effective metrics in plasma flows. One of the advantages of considering plasma slab in laboratory in the effective gravity is that in astrophysical settings the Reynolds number are very high as $Rm^{-1} \approx \eta \nabla^2 \approx \eta L^{-2} \approx \eta \times 10^{-20} cm^{-2}$ for for example a solar loop scale length of $10^{10} cm$, diffusion effects neglected. The paper is organized as follows: In section II a brief review on holonomic Frenet frame is presented where it is demonstrated that stretching of the flow is in generally associated with compressibility.

In section III the dynamo equation is solved to yield an effective metric and a marginal dynamo is found. In section IV slow dynamos are obtained. Section V presents a further example of the effective plasma in the background of acoustic black holes. Section VI addresses the conclusions and discussions are given.

II Stretching flows in Frenet frame

This section deals with a brief review of the Serret-Frenet holonomic frame [6] equations that are specially useful in the investigation of fast dynamos in magnetohydrodynamics (MHD) with magnetic diffusion. Here the Frenet frame is attached along the magnetic flow streamlines which possesses Frenet torsion and curvature [6], which completely determine topologically the filaments, one needs some dynamical relations from vector analysis and differential geometry of curves such as the Frenet frame $(\mathbf{t}, \mathbf{n}, \mathbf{b})$ equations

$$\mathbf{t}' = \kappa \mathbf{n} \quad (\text{II.1})$$

$$\mathbf{n}' = -\kappa \mathbf{t} + \tau \mathbf{b} \quad (\text{II.2})$$

$$\mathbf{b}' = -\tau \mathbf{n} \quad (\text{II.3})$$

The holonomic dynamical relations from vector analysis and differential geometry of curves by $(\mathbf{t}, \mathbf{n}, \mathbf{b})$ equations in terms of time

$$\dot{\mathbf{t}} = [\kappa' \mathbf{b} - \kappa \tau \mathbf{n}] \quad (\text{II.4})$$

$$\dot{\mathbf{n}} = \kappa \tau \mathbf{t} \quad (\text{II.5})$$

$$\dot{\mathbf{b}} = -\kappa' \mathbf{t} \quad (\text{II.6})$$

along with the flow derivative

$$\dot{\mathbf{t}} = \partial_t \mathbf{t} + (\vec{v} \cdot \nabla) \mathbf{t} \quad (\text{II.7})$$

From these equations and the generic flow

$$\dot{\mathbf{X}} = v_s \mathbf{t} + v_n \mathbf{n} + v_b \mathbf{b} \quad (\text{II.8})$$

one obtains

$$\frac{\partial l}{\partial t} = (-\kappa v_n + v_s') l \quad (\text{II.9})$$

where l is given by

$$l := (\mathbf{X}' \cdot \mathbf{X}')^{\frac{1}{2}} \quad (\text{II.10})$$

which shows that if v_s is constant, which fulfills the solenoidal incompressible flow

$$\nabla \cdot \mathbf{v} = 0 \quad (\text{II.11})$$

since v_n vanishes along the tube, one should have a non-stretched flow. This is exactly the choice $\mathbf{v} = v_0 \mathbf{t}$, where $v_0 = \text{constant}$ is the steady flow one uses here. The solution

$$\mathbf{B} = \nabla \phi \quad (\text{II.12})$$

shall be considered here. This definition of magnetic filaments is shows from the solenoidal carachter of the magnetic field

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{II.13})$$

III Acoustic black holes in marginal dynamo plasmas

Let us consider the non-linear dynamo flow equations [7]

$$[\partial_t - Rm^{-1} \nabla^2] \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad (\text{III.14})$$

and the coupled dynamo flow equation

$$[\partial_t - Re^{-1} \nabla^2] \mathbf{v} = -\nabla p + \mathbf{J} \times \mathbf{B} \quad (\text{III.15})$$

where Re^{-1} and Rm^{-1} are respectively the fluid and magnetic Reynolds numbers and \mathbf{J} is the magnetic current. Since the Reynolds flow numbers are finite, the magnetic field is not necessarily frozen in and the flow may possess magnetostatic fields. In the magnetostatic case one has

$$\mathbf{B} = \nabla \phi \quad (\text{III.16})$$

and to obtain the effective metric as usual one considers the irrotational flow [7]

$$\mathbf{v} = \nabla \psi \quad (\text{III.17})$$

First the magnetostatic equation decouples both equations above since the current $\mathbf{J} = \nabla \times \mathbf{B}$ vanishes. Along the other magnetohydrodynamic equations

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (\text{III.18})$$

and the barotropic equation of state

$$p = p(\rho) \quad (\text{III.19})$$

where p is the pressure. Now by considering the pressure and scalar fluctuations [7]

$$p = p_0 + \epsilon p_1 \quad (\text{III.20})$$

$$\psi = \psi_0 + \epsilon \psi_1 \quad (\text{III.21})$$

$$\rho = \rho_0 + \epsilon \rho_1 \quad (\text{III.22})$$

Substitution of these fluctuations into the evolution flow equations

$$\partial_t [c_{\text{sound}}^{-2} \rho_0 (\partial_t \psi_1 + \mathbf{v}_0 \cdot \nabla \psi_1 - Re^{-1} \nabla^2 \psi_1)] = \nabla \cdot [\rho_0 \nabla \psi_1 - c_{\text{sound}}^{-2} \rho_0 \mathbf{v}_0 (\partial_t \psi_1 + \mathbf{v}_0 \cdot \nabla \psi_1)] \quad (\text{III.23})$$

which yields the equation

$$\frac{1}{\sqrt{-g}} [\partial_\mu (\sqrt{-g} g^{\mu\nu} g^{\mu\nu} \partial_\nu \psi_1)] = Re^{-1} \nabla^2 \partial_t \psi_1 \quad (\text{III.24})$$

where $\mu = 0, 1, 2, 3$, are the spacetime of effective gravity. Here one also uses the relation $c_{\text{sound}} = \frac{\partial p}{\partial \rho}$. Now let us note that the term on the RHS of this equation can be expressed as

$$\nabla^2 \partial_t \psi_1 = \nabla \cdot \partial_t \nabla \psi_1 = \partial_t (\nabla \cdot \mathbf{v}_1) \quad (\text{III.25})$$

Now note that when the flow is incompressible $\nabla \cdot \mathbf{v} = 0$, or non-stretching this equation reduces to

$$\partial_t \frac{1}{\sqrt{-g}} [\partial_\mu (\sqrt{-g} g^{\mu\nu} g^{\mu\nu} \partial_\nu \psi_1)] = 0 \quad (\text{III.26})$$

Therefore in the magnetostatic case the decoupling of effective black hole in marginal dynamos can be easily obtained. These dynamo like acoustic black holes can be called linear. In the next section one presents the effective acoustic black holes are present in non-linear slow dynamos.

IV Kerr-Schild effective slow dynamos spacetime

In this section one shall consider the effect of the Riemannian acoustic black hole metric may have on non-linear dynamos in plasmas, where the back reaction of Lorentz magnetic force cannot be neglected as in last section. The presence of plasma jets in supermassive black holes , is more than enough motivation for the investigation of the presence of analogue models in plasmas, as discussed here. This time the dynamo flow coupling equations cannot be decoupled anymore. Let us now consider the nonlinear dynamo equation in the form

$$[\partial_t - Rm^{-1}\nabla^2]\mathbf{B} = (\mathbf{v}.\nabla)\mathbf{B} - (\mathbf{B}.\nabla)\mathbf{v} \quad (\text{IV.27})$$

Let us now show that the acoustic black hole metric given by $g^{00} = -\frac{1}{c_{sound}^2\rho_0}$, $g^{0j} = -\frac{1}{c_{sound}^2\rho_0}v_0^j$ and $g^{ij} = \frac{1}{c_{sound}^2\rho_0}(c_{sound}(\delta^{ij} - v_0^i v_0^j))$ and the magnetic field perturbation $\mathbf{B} = \mathbf{B}_0 + \epsilon\mathbf{B}_1$ where $\epsilon \ll 1$ implies that the coupled dynamo flow equation becomes

$$[\mathbf{Re}\gamma + i\mathbf{Im}\gamma + Re^{-1}K^2]\mathbf{B}_0 = i(\nabla\psi_0.\nabla(\mathbf{K}.\mathbf{x}))\mathbf{B}_0 \quad (\text{IV.28})$$

where $\mathbf{x} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is the vector describing points in the rectangular coordinates of the slab and \mathbf{K} is the wave vector of the ansatz for the magnetic field

$$\mathbf{B}_0 = \exp[\gamma t + i(\mathbf{K}.\mathbf{x})]\mathbf{b}_0 \quad (\text{IV.29})$$

and from the equation $div\mathbf{B} = 0$ one obtains $\mathbf{B}_0.\mathbf{K} = 0$ which in turn yields

$$(\partial_x + \partial_y)\mathbf{v}_0 = 0 \quad (\text{IV.30})$$

where

$$\mathbf{B}_0 = \exp[\gamma t + i(\mathbf{K}.\mathbf{x})]\mathbf{b}_0 \quad (\text{IV.31})$$

Thus from the acoustic metric one obtains finally an expression for the metric in terms of the dynamo growth rate as

$$\mathbf{Im}\gamma = (\nabla\psi_0.\nabla(\phi)) \quad (\text{IV.32})$$

$$\mathbf{Re}\gamma = -Re^{-1}K^2 \quad (\text{IV.33})$$

Note from (IV.33) that the dynamo is not at all fast since $\mathbf{Re}\gamma < 0$. Actually since the imaginary part of the magnetic field

$$\mathbf{Im}\mathbf{B}_0 = \mathbf{b}_0 \sin[\mathbf{Im}\gamma t + \phi] \quad (\text{IV.34})$$

Here $\phi = \mathbf{K} \cdot \mathbf{x}$ is the magnetic phase. Thus from Equation (IV.32) is given by

$$v^{0j} \partial_j \phi = v^{0j} K_j \quad (\text{IV.35})$$

the acoustic metric one obtains finally an expression for the metric in terms of the dynamo growth rate as

$$g^{0j} = c_{sound} \rho_0 \frac{\mathbf{Im} \gamma}{K^2} K^j \quad (\text{IV.36})$$

where $K^2 = K_j K^j$. The other acoustic black hole dynamo metric are

$$g^{ij} = \frac{1}{\rho_0} c_{sound}^{-2} (c_{sound} \delta^{ij} - [Im \gamma]^2 \frac{\partial^i \phi \partial^j \phi}{K^4}) \quad (\text{IV.37})$$

Note that when one considers planar magnetic waves in plasmas or dynamo waves where the wave vector \mathbf{K} is constant one obtains the Kerr-Schild spacetime metric

$$g^{ij} = \frac{1}{\rho_0} c_{sound}^{-2} (\delta^{ij} - [Im \gamma]^2 \frac{K^i K^j}{K^4}) \quad (\text{IV.38})$$

This completes our task of showing that an artificial acoustic black hole can be obtained in the effective plasma slow dynamo spacetime. Metric (IV.38) resembles metrics in scalar gravity and is definitely a spacetime carrying the growth of magnetic field information from $Im \gamma$ term. Actually K^j is the Killing vector of the Kerr-Schild stationary metric symmetry so much useful in general relativistic metrics.

V Acoustic black holes in collision plasmas

In this section a further physical example is given of the Einstein's effective gravity in plasmas dissipative media. Let us then to consider the equation including the dissipation effective and the electric field as

$$\rho[\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v}] = \pm en \mathbf{E} - \rho \nu \mathbf{v} - \nabla p \quad (\text{V.39})$$

here \mathbf{E} is the electric field and ν is the viscosity coefficient. From the same reasoning of the last section one has

$$\partial_t [c_{\text{sound}}^{-2} \rho_0 (\partial_t \psi_1 + \mathbf{v}_0 \cdot \nabla \psi_1 - R e^{-1} \nabla^2 \psi_1)] \pm en \phi_1 - \rho \nu \psi_1 - p_1 = \nabla \cdot [\rho_0 \nabla \psi_1 - c_{\text{sound}}^{-2} \rho_0 \mathbf{v}_0 (\partial_t \psi_1 + \mathbf{v}_0 \cdot \nabla \psi_1)] \quad (\text{V.40})$$

which yields the equation

$$\frac{1}{\sqrt{-g}} [\partial_\mu (\sqrt{-g} g^{\mu\nu} g^{\mu\nu} \partial_\nu \psi_1)] = \pm en \phi_1 - \rho \nu \psi_1 \quad (\text{V.41})$$

where $\mu = 0, 1, 2, 3$, are the spacetime of effective gravity. Here one also uses the relation $c_{\text{sound}} = \frac{\partial p}{\partial \rho}$. Now let us note that by expressing ψ_1 as

$$\psi_1 = e^{\omega t + i(k_r r)} \quad (\text{V.42})$$

and by considering that the background effective (2+1)-D spacetime Riemannian line element is given by Visser fluid [8] metric

$$ds^2 = -c_{\text{sound}}^2 dt^2 - (dr - \frac{A}{r} dt)^2 + (rd\theta - \frac{B}{r} dt)^2 \quad (\text{V.43})$$

one obtains the following dispersion relation

$$[\rho_0(\omega - \nu) + \frac{AK_0}{r}] \psi_1 = \pm en \phi_1 - p_1 \quad (\text{V.44})$$

From the base part of the fluctuation equation

$$\pm en \phi_0 = [\rho_0(\omega + \nu) - \frac{A}{r} K_r] \psi_0 \quad (\text{V.45})$$

Since by the equations one has

$$\psi_0 = -A \ln r \quad (\text{V.46})$$

Substitution of this expression into (V.45) allows us to determine the electric potential of the collision plasma

$$\pm en\phi_0 = -A[\rho_0(\omega + \nu) - \frac{A}{r}K_r]lnr \quad (\text{V.47})$$

This and other interesting physical properties of this plasma effective spacetime may be considered elsewhere.

VI Conclusions

A further example of an artificial Riemannian black hole in the Euclidean geometry of plasma dynamo slabs in rectangular coordinates, is added to innumerable examples of artificial optical and electromagnetic effective spacetimes [3]. Other geometries can be considered elsewhere which come from plasma physics as the construction of the artificial black holes in toroidal magnetic geometry of tokamaks and stellarators. The toric geometry has actually been considered by Garay [8] in the investigation of Bose-Einstein condensates. An interesting example of an artificial magnetic metric seems to have been given by Titov [9] where he considers the case of covariant formulation of solar loops as well but certainly with different details and motivation. Actually Titov's effective metric is more similar to Novello's, in reference [8], dielectric nonlinear metric $g^{\mu\nu} = \epsilon\eta^{\mu\nu} - \frac{\epsilon'}{E}(E^\mu E^\nu - E^2 v^\mu v^\nu)$ where E^ν is the electric field and ϵ is electric permittivity and $\epsilon' = \frac{d}{dE}\epsilon$. This metric and Titov's possess the Kerr-Schild form for rotating black holes. To resume since general relativistic plasma physics [10] in black hole physics is well established along with the presence of sound waves in plasmas, the presence of acoustic black holes is naturally investigated in this paper.

VII Acknowledgements

I thank financial supports from Universidade do Estado do Rio de Janeiro (UERJ) and CNPq (Brazilian Ministry of Science and Technology).

References

- [1] Science **319**,1367 (2008).
- [2] G Volovik, **Universe in a Helium droplet** (2003) Oxford University Press.
- [3] S. Childress, A. Gilbert, **Stretch, Twist and Fold: The Fast Dynamo** (1996),Springer, Berlin.
- [4] L C Garcia de Andrade, Phys Rev **D 70** (2004).
- [5] L C Garcia de Andrade, Phys Lett A (2005).
- [6] L. C. Garcia de Andrade, Physics of Plasmas 13, 022309 (2006).
- [7] L. C. Garcia de Andrade, Physics of Plasmas 14 (2007). L.C. Garcia de Andrade, Non-holonomic dynamo filaments as Arnolds map in Riemannian space, Astronomical notes (2008) in press. L.C. Garcia de Andrade, Physica Scripta 13 (2006).
- [8] L Garay,Acoustic black holes in dilute Bose-Einstein condensates, in **Artificial Black Holes** Ed M Novello, M Visser and G Volovik.
- [9] S Titov, Generalizes squashing factor for covariant description of magnetic in the solar corona, astro-ph/07030617v1.
- [10] T Tajima and K Shibata, Plasma astrophysics, (1997) Addison-Wesley.